# Math 8

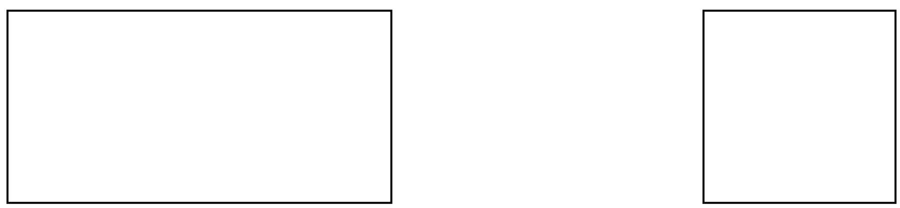
## Unit I – Square Roots and Pythagorean Theorem

### Notes

***1.1 – Square Numbers and Area Models***

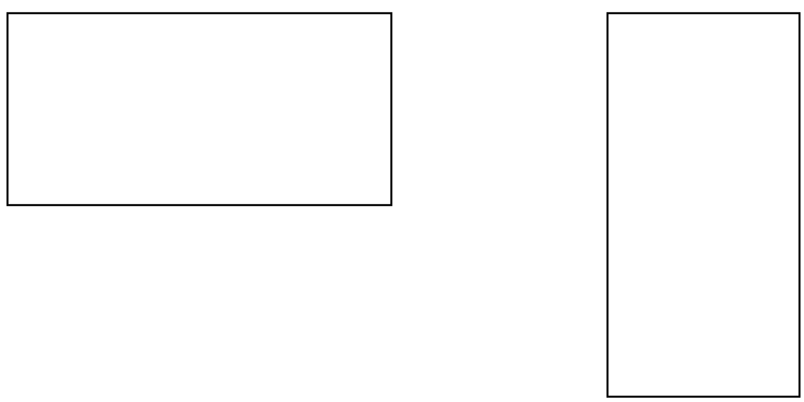
- How are squares and rectangles similar and different?
- Both are quadrilaterals (four sides) and each contains 4 right angles (right angle = 90º)
- However, in a square, all sides are the same length. In a rectangle, there are two sides that match.

  *Example:*



Rectangle Square

- Are the following shapes the same or different?



- ***Congruent*** – when shapes exactly match, but have different orientation.
- Recall the definition of ***area***: the number of square units needed to cover a region.
- Recall the definition of ***perimeter***: the distance around a closed shape.

  ****Do the investigation (investigate) on page 6. We can work with your table partner if you wish.****

- When we multiply a number by itself, we square the number.

  Example: The square of 6 is 6 x 6 = 36

  What's the square of 8? ________________________

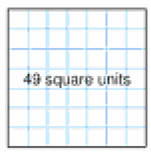
- We say that 6 squared is 36. We say that 8 squared is __________.
- 36 is a ***square number***, or a ***perfect square***.
- One way to model a square number is to draw a square whose area is equal to the square number.
- To figure out the number that 'squared' (times by itself), simply find the length of one side (on the square).

**Example 1**

Show that 49 is a square number.
Use a diagram, symbols, and words.

**A Solution**

Draw a square with area 49 square units.
The side length of the square is
Then, 49 = × =
We say: Forty-nine is squared.



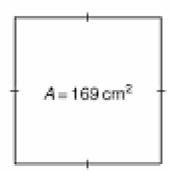

How about this one…it's tricky licky!!!!

**Example 2**

A square picture has area 169 cm$^2$.
Find the perimeter of the picture.

**A Solution**

The picture is a square with area 169 cm$^2$.
Find the side length of the square:




Find a number which, when multiplied by itself, gives 169.

So, the picture has side length

Perimeter is the distance around the picture.
So, $P$ = cm + cm + cm + cm
= cm
The perimeter of the picture is cm.

- Is 1 a square number? How do you know?

- Suppose you know the area of a square. How can you find its perimeter?

- Suppose you know the perimeter of a square, how do you find its area?

***Do the following questions: Pgs 8 and 9, Questions #**4-7,10-13,16,19*****

### *1.2 – Squares and Square Roots*

- A factor is a number that divides evenly into another number.

*Example:* What are the factors of 6? (All the number that can divide evenly into 6)

What are the factors of 16?

******A factor that occurs twice is only written once in the list of factors******

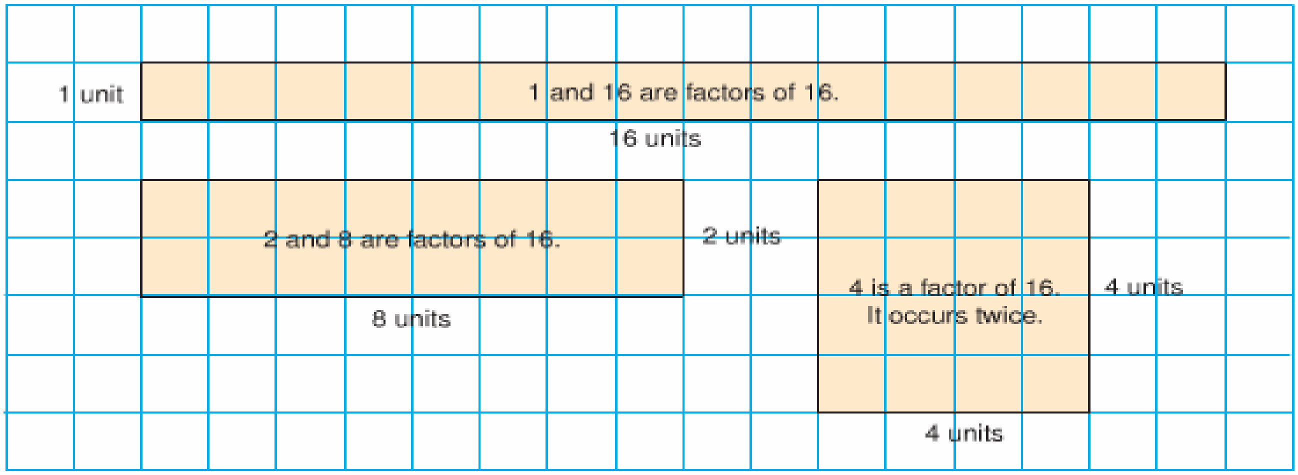
- Do the investigation (investigate) on page 11 using the chart provided.

  - ➤ Which numbers have only two factors? What do you notice about these numbers?
  - ➤ Which numbers have an even number of factors, but more than 2 factors?
  - ➤ Which numbers have an odd number of factors?

Compare your answers with your table partner. Which numbers in the chart are square numbers? How do you know?

What seems to be true about factors of square numbers?

We can also use factoring.
Factors of a number occur in pairs.
These are the dimensions of a rectangle.




Sixteen has 5 factors: 1, 2, 4, 8, 16

Since there is an odd number of factors, one rectangle is a square.
The square has side length 4 units.
We say that 4 is a **square root** of 16.
We write: $4 = \sqrt{16}$

**A factor that occurs twice is only written once in the list of factors.**

When a number has an odd number of factors, it is a square number

- When a number is multiplied by itself, we ***square*** the number.
- The opposite of squaring, is ***square root***.
- Squaring and taking the square roots are inverse operations. That is, they undo each other.

$4 \times 4 = 16$
so, $4^2 = 16$

$\sqrt{16} = \sqrt{4 \times 4} = \sqrt{4^2}$
$= 4$

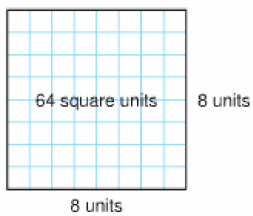
- Let's do some examples:

  a. Find the square of each number: 3 and 7

b. Find the square root of 64.

Two ways to solve this one, you can use grid paper or factors

Use grid paper.
Draw a square with area 64 square units.
The side length of the square is 8 units.
So, $\sqrt{64} = 8$




***Example 2***
***Another Solution***

Find pairs of factors of 64.
Use division facts.

| | |
|---|---|
| $64 \div 1 = 64$ | 1 and 64 are factors. |
| $64 \div 2 = 32$ | 2 and 32 are factors. |
| $64 \div 4 = 16$ | 4 and 16 are factors. |
| $64 \div 8 = 8$ | 8 is a factor. It occurs twice. |

The factors of 64 are: 1, 2, 4, **8**, 16, 32, 64
A square root of 64 is 8, the factor that occurs twice.

c. Is 110 a square number? See if you can prove it…I dare you!!!!

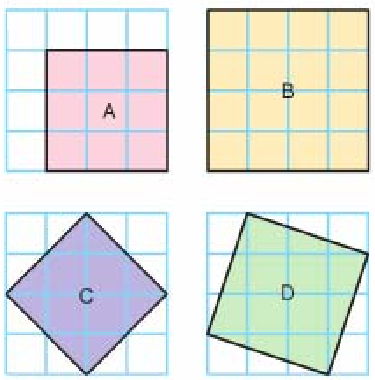
**Do the following questions: Pgs 15 and 16, Questions #**5-7,9,11-13,15,19,22***

### *1.3 – Measuring Line Segments*

- Do the investigation (investigate) on page 17. Work with your table mate if you wish. Use the supplied grid paper.
- I have scanned the investigation below. Please answer the questions as well.

**Investigate**

Work with a partner. You will need 1-cm grid paper.
Copy the squares below.
Without using a ruler, find the area and side length of each square.




What other squares can you draw on a 4 by 4 grid?
Find the area and side length of each square.
Write all your measurements in a table.

**Reflect & Share**

How many squares did you draw?
Describe any patterns in your measurements.
How did you find the area and side length of each square?
How did you write the side lengths of squares C and D?

- Remember that area of a square = length x length or side x side
- Can show this formula as follows:

Area of a square = length x length = $(\text{length})^2$

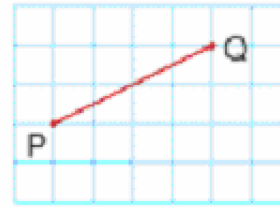
If the side length is $l$, then the area is $l^2$

When the area is A, the side length is $\sqrt{A}$

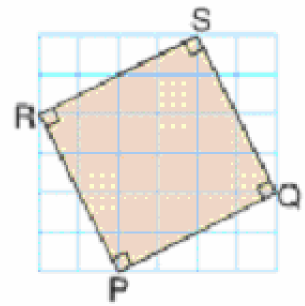
- Based on this, we can calculate the side length of any line by thinking of it as the length of a square.

- Try this example:

Find the length of line segment PQ.

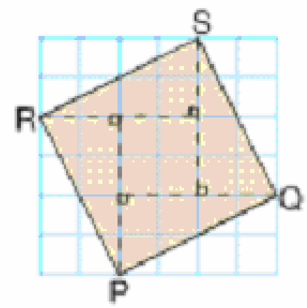



### *A Solution*




Use a straightedge and protractor to construct a square on line segment PQ. Then, the length of the line segment is the square root of the area.

Cut the square into 4 congruent triangles and a smaller square.

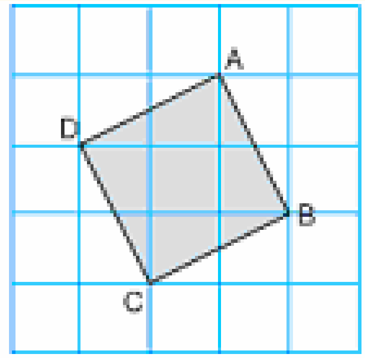



The area of each triangle is:

- Try this example…it is another method to find area

a) Find the area of square ABCD.

b) What is the side length AB of the square?

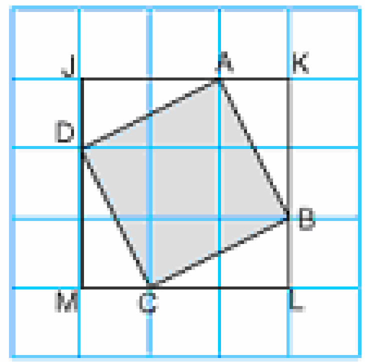



### A Solution

a) Draw an enclosing square JKLM.

The area of JKLM $= 3^2$ square units

$= 9$ square units




The triangles formed by the enclosing square are congruent.

Each triangle has area:

****Do the following questions: Pgs 20 and 21, Questions #**1-7 9, 10******

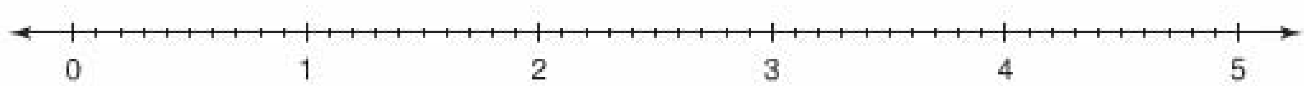
### *1.4 – Estimating Square Roots*

- Let's recap what you know (hopefully ☺)
- A perfect square is a number, that when square rooted, gives an integer.
  For example: $\sqrt{9} = \sqrt{3 \times 3}$
  $= 3$
- Square root of a number is the side length of a square with an area equal to that number.
- How would you go about estimating a square root value? What strategies would you use?

- Think of all the perfect squares, 1, 4, 9, 16, 25, 36 etc. Use them to help you estimate a square root that is not a perfect square.
- Turn to page 22 in your book. We are going to do the Investigation together.

**Investigate**

Work with a partner.
Use a copy of the number line below.
Place each square root on the number line to show its approximate value: $\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$, $\sqrt{18}$, $\sqrt{24}$
Write each estimated square root as a decimal.
Use grid paper if it helps.




- What strategies did we use?

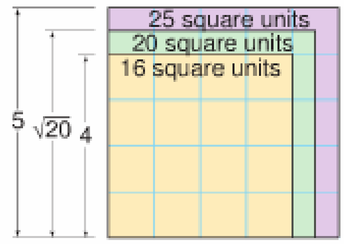
- How could we check our estimations

- Let's look at what the book has to say…

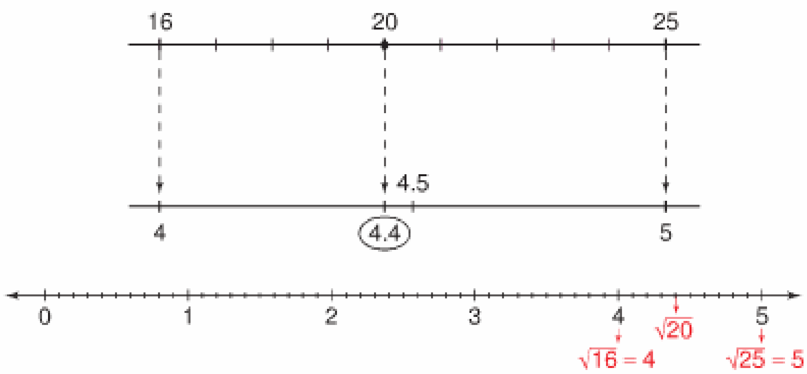
Here is one way to estimate the value of $\sqrt{20}$ :

- 25 is the square number closest to 20, but greater than 20.
  On grid paper, draw a square with area 25.
  Its side length is: $\sqrt{25} = 5$
- 16 is the square number closest to 20, but less than 20.
  Draw a square with area 16.
  Its side length is: $\sqrt{16} = 4$

Draw the squares so they overlap.




A square with area 20 lies between these two squares.
Its side length is $\sqrt{20}$.
20 is between 16 and 25, but closer to 16.
So, $\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$, but closer to $\sqrt{16}$.
So, $\sqrt{20}$ is between 4 and 5, but closer to 4.
An estimate of $\sqrt{20}$ is 4.4 to one decimal place.




- Based on the above explanation and the investigation, how would you estimate √76?

- How about this example…

  A square garden has an area of 139 $m^2$.

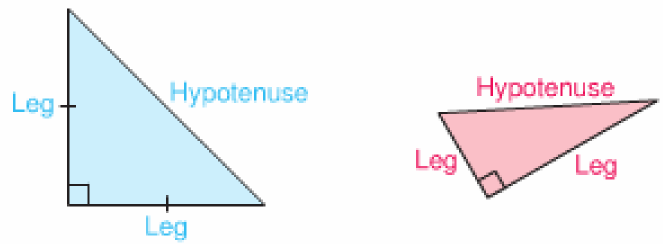
  1. What are the approximate dimensions of the garden to two decimal places?

  2. Wire fence is needed to keep out ravenous rabbits and rabid raccoons. About how much fencing would be needed around the garden?

****Do the following questions: Pgs 20 and 21, Questions #**1-7 9, 10******

## *1.5 – The Pythagorean Theorem*

- Before we jump into the Pythagorean Theorem, we need to learn a bit about right angle triangles.

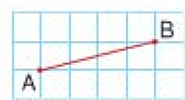



**Isosceles right triangle** **Scalene right triangle**

- Ok, now let's go to the investigation…let's see if you can connect the dots ☺ Turn to page 31 and let the fun begin.

### Investigate

Work on your own.
You will need grid paper, centimetre cubes, and a protractor.



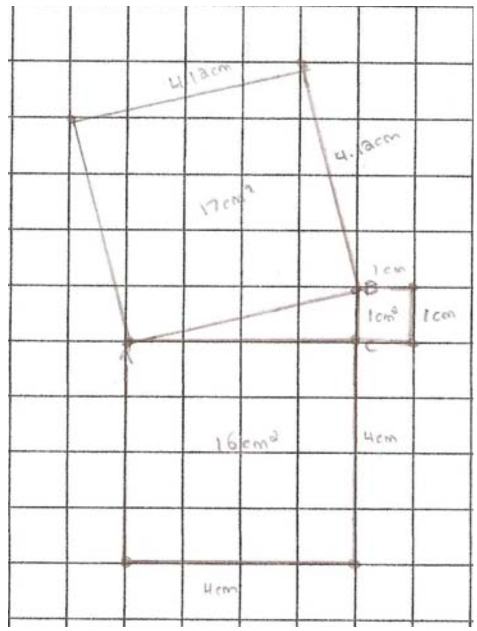

➤ Copy line segment AB.
Draw right triangle ABC that has segment AB as its hypotenuse.
Draw a square on each side of △ABC.
Find the area and side length of each square.

➤ Draw 3 different right triangles, with a square on each side.
Find the area and side length of each square.
Record your results in a table.

See page 18 if you have forgotten how to find the area of a square on a line segment.

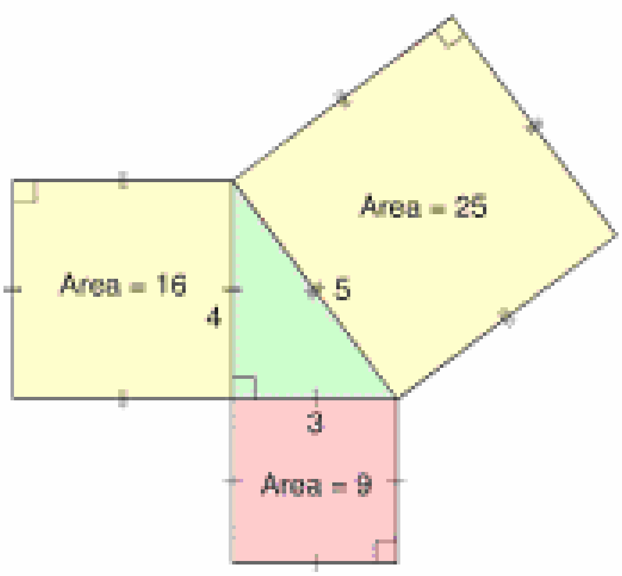
| | Area of Square on Leg 1 | Length of Leg 1 | Area of Square on Leg 2 | Length of Leg 2 | Area of Square on Hypotenuse | Length of Hypotenuse |
|---|---|---|---|---|---|---|
| Triangle ABC | | | | | | |
| Triangle 1 | | | | | | |
| Triangle 2 | | | | | | |
| Triangle 3 | | | | | | |

This is what the first triangle should look like.




- Based on the above picture, try and make three more right angled triangles and fill in the table above.
- Look at this picture from your textbook…notice anything about the areas?

Here is a right triangle, with a square drawn on each side.




The area of the square on the hypotenuse is 25.
The areas of the squares on the legs are 9 and 16.

- Hopefully you notice that the area of the legs adds up to the area of the hypotenuse. This is called the Pythagorean Theorem.
- This relationship is true for ALL right angle triangles. Go back and check over the examples in the investigation. Do you see the same relationship?
- The Pythagorean Theorem allows us to find the length of any side of a right angle triangle, if we know the other two sides.
- When doing problems with the Pythagorean Theorem you need to make sure you have the right angle triangle labeled properly. Make sure you know which side are the ***legs*** and which side is the ***hypotenuse***.
- The Pythagorean Theorem can be shown in an equation.

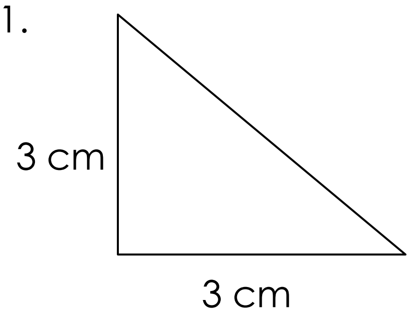
$$a^2 + b^2 = h^2$$

a and b = the legs h = the hypotenuse

*If you square the lengths of the legs and add them together, this equals the square of the hypotenuse length.*

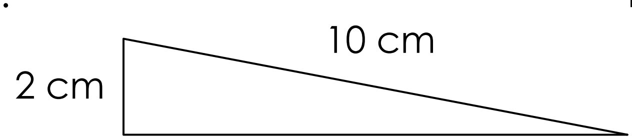
- Alright, enough of this talk…let's do some examples ☺

1.



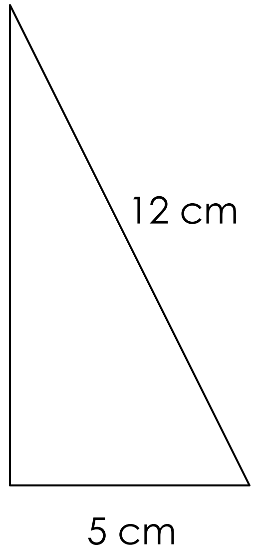

Find the length of the hypotenuse. Give the length to two decimal places.

2.




Find the length of the missing leg. Give the length to two decimal places.

3.

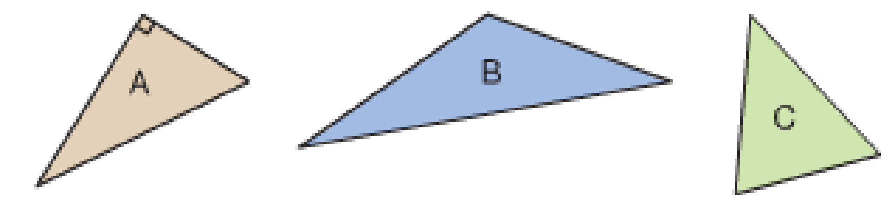



Find the length of the missing leg. Give the answer to two decimal places.

- Ok young grasshoppers, do out and explore the Pythagorean Theorem.
- No book questions this time…I'll give you a handout ☺

## *1.6 – Exploring the Pythagorean Theorem*

- Look at the triangles below. Which is a right angle triangle? Which is an acute triangle? Lastly, which is an obtuse triangle?




Here are some helpful definitions:

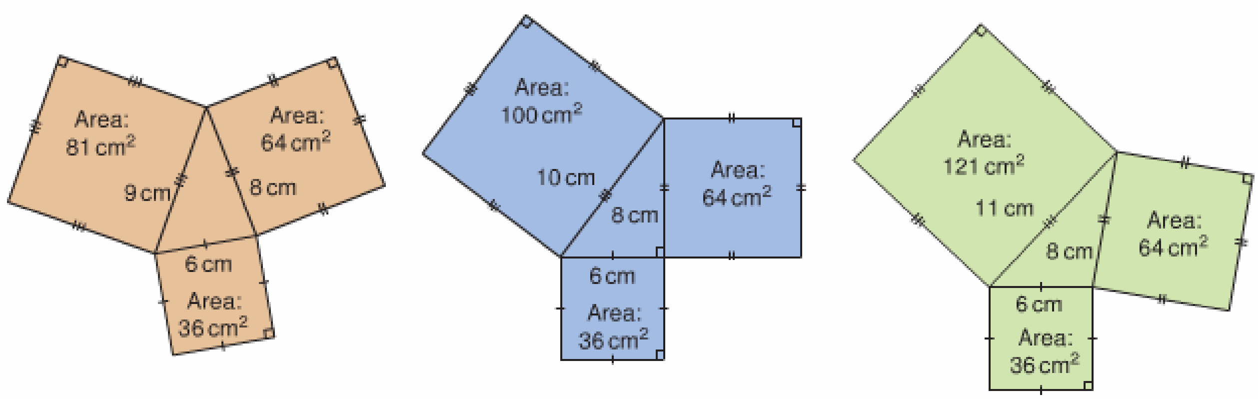
*Obtuse triangle*: A triangle with one angle that is bigger than 90º.

*Right triangle*: triangle that has one right angle

*Acute triangle*: Triangle with 3 acute angles (acute angle=less than 90º)

- Let's take this further…look below

Here are an acute triangle, a right triangle, and an obtuse triangle, with squares drawn on the sides of each triangle.



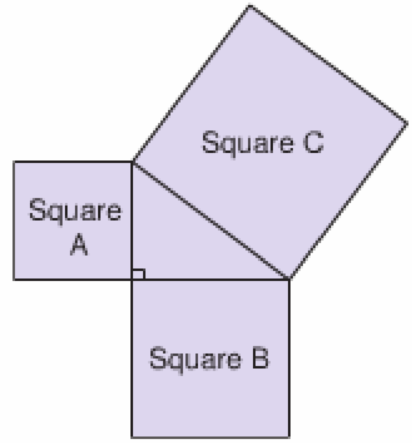

For each triangle, compare the sum of the areas of the two smaller squares to the area of the largest square.

| Triangle | Sum of Areas of Two Smaller Squares | Area of Largest Square |
|---|---|---|
| Acute | $36 + 64 = 100$ | 81 |
| Right | $36 + 64 = 100$ | 100 |
| Obtuse | $36 + 64 = 100$ | 121 |

Notice that the Pythagorean Theorem is true for the right triangle only.

We can use these results to identify whether a triangle is a right triangle.
If Area of square A + Area of square B = Area of square C,
then the triangle is a right triangle.

If Area of square A + Area of square B ≠ Area of square C,
then the triangle is not a right triangle.




**The symbol ≠ means "does not equal."**

- If you are just given the side lengths of a triangle, this above relationship allows you to determine if it is a right angle triangle.

- Try these examples:

  Determine whether each triangle with the given side lengths is a right angle triangle.

  a.) 5 cm, 5cm, 8 cm b.) 7 cm, 24 cm, 25 cm

  (Hint: sketch the triangles and draw the squares on each side. Find the areas and see if they stratify the Pythagorean Theorem)

- A set of 3 numbers that satisfies the Pythagorean Theorem is called a Pythagorean triple.

  *For example*: 3,4 and 5 are a Pythagorean triple because $3^2 + 4^2 = 5^2$

- Does any of the examples above qualify as a Pythagorean triple?
- If you need to check if a set of numbers is a Pythagorean triple, simply put them into the Pythagorean Theorem equation and see if it works out.

- Try these fabulous examples:

  1. 8, 15 and 18

  2. 11, 60 and 61

****Do the following questions: Pgs 43 and 44, Questions #**3-6(a-e),8******

## *1.7 – Applying the Pythagorean Theorem (Word Problems!!!)*

- Ok, once again we are faced with word problems involving Pythagorean Theorem.
- Remember the formula,

$$a^2 + b^2 = h^2$$

a and b = the legs    h = the hypotenuse

- Draw pictures!!!1 They are so helpful. Pythagorean Theorem is based on right angle triangles, so use the information in the question and draw them. It will be WAY easier to solve.
- Ok, take a deep cleansing math breathe…here we go ☺

Examples:

1.

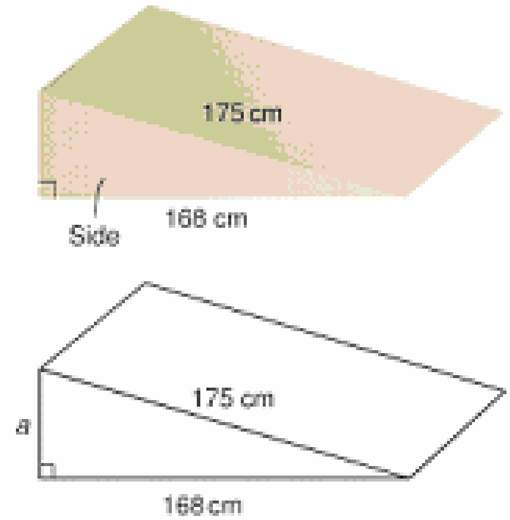
Marina helped her dad build a small rectangular table for her bedroom.
The tabletop has length 56 cm and width 33 cm.
The diagonal of the tabletop measures 60 cm.
Does the tabletop have square corners?
How do you know?

**A square corner is a right angle.**

First step, draw a picture. Decide what you need to find and then go for it!

2.

A ramp is used to load a snow machine onto a trailer.
The ramp has horizontal length 168 cm
and sloping length 175 cm.
The side view is a right triangle.
How high is the ramp?




***Do the assigned handout. The unit is all done!!! Time for a test ☹***