Math 8
Unit II – Integers
Notes

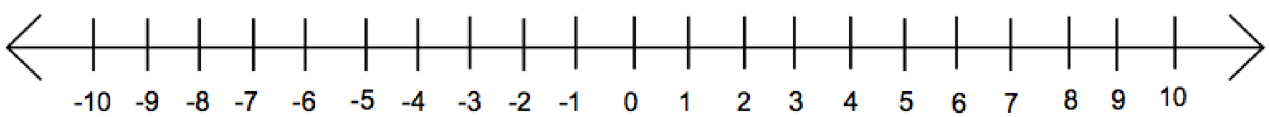
Review of the Basics

- Numbers such as -10, +15,000, -2, and +29 are called ***Integers.***
- If an integer is ***greater*** than zero, we say he sign is *positive*.
- If an integer is ***less than*** zero, we say the sign is *negative*.
- +1, +2, +3,…are positive integers. They *MAY* be written without the sign.
- -1, -2, -3,… are negative integers. They are ***NEVER*** written without a sign.
- For each positive integer, there is a negative integer and this integer is called an ***opposite***.
- Negative integers are the opposite of positive integers.

-3 and +3 are opposites so are -7 and +7

Do ***Write and Integer Worksheet***

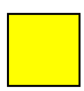
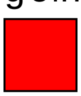
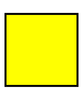
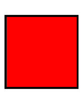
- How do we compare integers? One way is to use a number line.

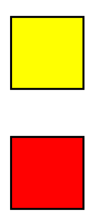


- A number line is labeled with the integers increasing in order from *left to right*. It continues to extent in both directions.
- For any two different places on the number line, the integer on the ***right is greater*** than the integer on the left.

Do ***Comparing Integers Worksheet***

Representing Integers

- We are going to use yellow tiles  to represent positive integers and red tiles  to represent negative integers.
- One yellow tile  will represent +1
- One red tile  will represent -1
- A red ***AND*** yellow tile combine to model 0 (zero)

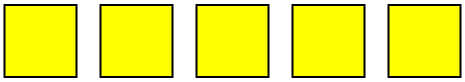


+1 We call this a ***zero pair.***

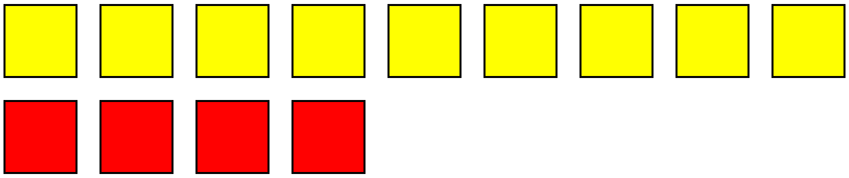
-1

- How can we model +5?

  There are numerous different ways. How about…



= +5



= +5

  Can you think of any more?

- See if you can model -4 at least three different ways ☺

- Model each of the following in at least two different ways:

  1. +7 2. -2 3. -8

Adding Integers

- We are going to cover two different methods. Adding integers using tiles and also using the number line.

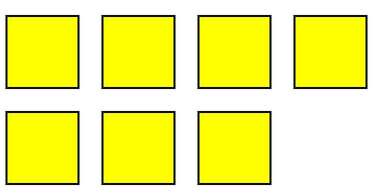
Tile Method

- To add integers using tiles, model each integer with the appropriate tile. Combine the tiles. ***Remove*** any zero pairs and whatever is left, is the answer. *Easy breezy lemon squeezy* ☺

Examples: Add using tiles

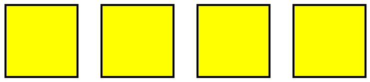
1. (+4) + (+3)

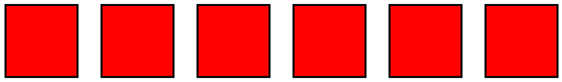
+4 =
+3 =



Combine the tiles. Any zero pairs?
Nope. Therefore, what's left is the answer
How many yellow tiles?
Answer =

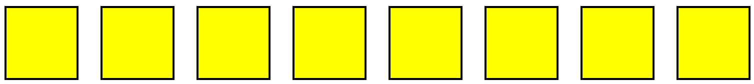
2. (+4) + (-6)

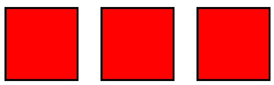
+4 = 

-6 = 

Combine the tiles. Any zero pairs? Get rid of the zero pairs. What's left over is the answer. ***Answer =***

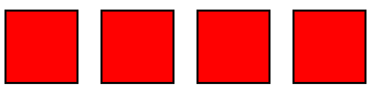
3. (+8) + (-3)

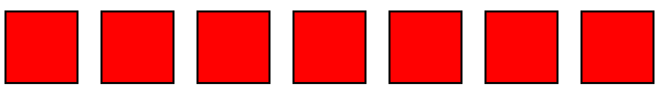
+8 = 

-3 = 

Combine the tiles. Any zero pairs? Get rid of the zero pairs. What's left is the answer. ***Answer =***

4. (-4) + (-7)

- 4 = 

- 7 = 

Combine the tiles. Any zero pairs? Get rid of the zero pairs. What's left is the answer. ***Answer =***

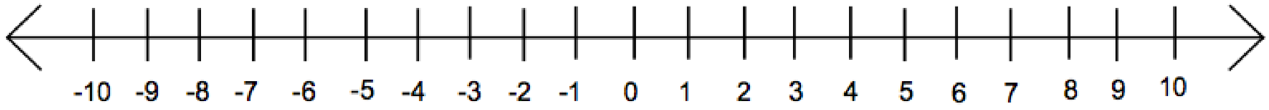
Number line method

- Two different ways this can be done.

  1st Method

  - Start at zero on the number line. Negative integers point left, positive integers point right.
  - From zero, move in the direction of the first integer. Then move in the direction of the second integer.
  - Where you end up, is the answer.
  - Ok, I know that's hard to follow, turn the page and let's try and example ☺

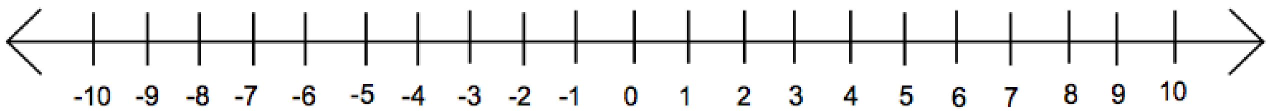
Example: (-5) + (+8)



2nd Method

- Very similar. Instead of starting at zero, start at the first integer and then go to the second integer (using the proper direction). Don't worry, example below ☺

Example: (-4) + (+9)



Do ***Adding Integers Worksheet***

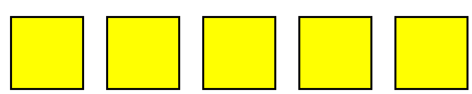
Subtracting Integers

- Just like adding integers, there are two different methods we will be covering to subtract. Subtracting using tiles and also using a number line.

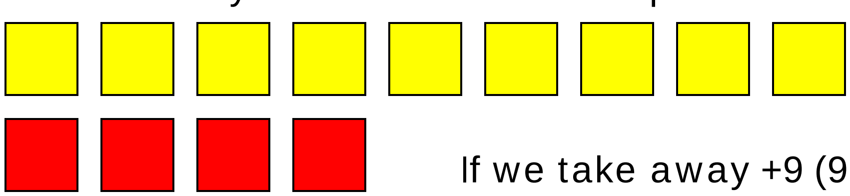
Tile method

- Model the first integer using tiles. Recall that yellow tiles = +1 and red tiles = -1.
- Take away the number of tiles indicated by the second integer.
- If you need more tiles, just add a zero pair (both a red and yellow tile). Because a zero pair equals zero, adding a zero pair has no affect on the equation. What is (+3) + 0 = ? Did adding the zero change anything?

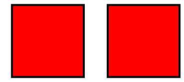
Examples: Subtract using tiles

1. (+5) – (+9) = Model (+5) = 

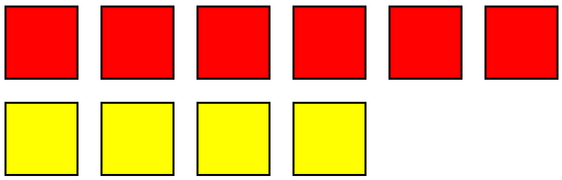
Not enough tiles to take away +9. In order to take away 9 we need to add 4 more yellow tiles. Add 4 zero pairs.



If we take away +9 (9 yellow tiles), what's left?

2. (-4) – (-6) = Model (-2) = 

Not enough tiles to take away -6. How many zero pairs do we have to add? We need to add ____ zero pairs.



What happens if we take away -6? What’s left? What’s the answer?

Try these ones:

(-9) – (+4) =

(+4) – (-5) =

(-3) – (-9) =

Number Line Method

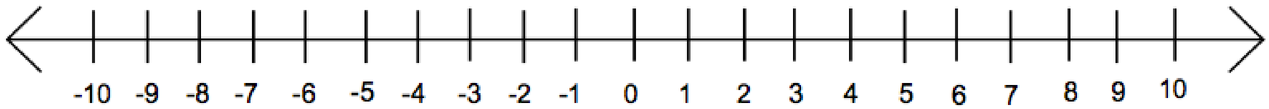
- To subtract an integer, we add the opposite. Keep the first integer the same, change the minus (-) to a (+) and change the 2nd integer to it’s opposite. So (-3) – (-6) would become (-3) + (+6).
- To add integers, recall you start at zero (or the first integer), move left if the integer is negative and right if the integer is positive.

Examples:

(+2) – (+9) =

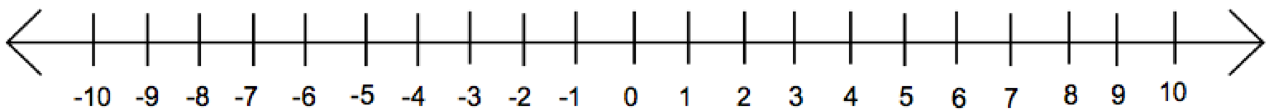
Change to addition. Keep the first integer the same, change to addition and change the 2nd integer to it's opposite. So in this case,

(+2) + (-9) =

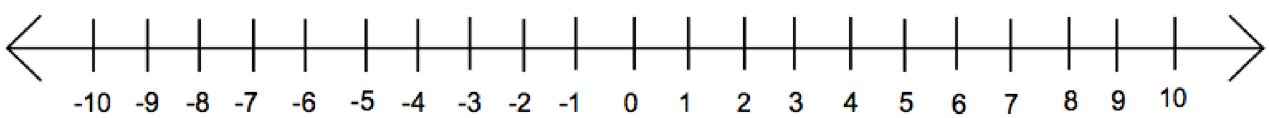


Try these ones:

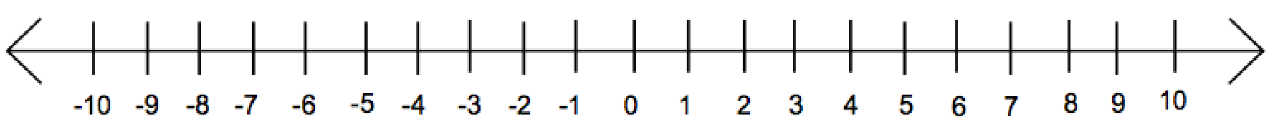
(-4) – (-8) =



(+9) – (+5) =



(-8) – (+7) =

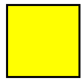
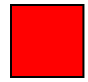


Do ***Subtracting Integers Worksheet**

Multiply Integers (Section 2.1)

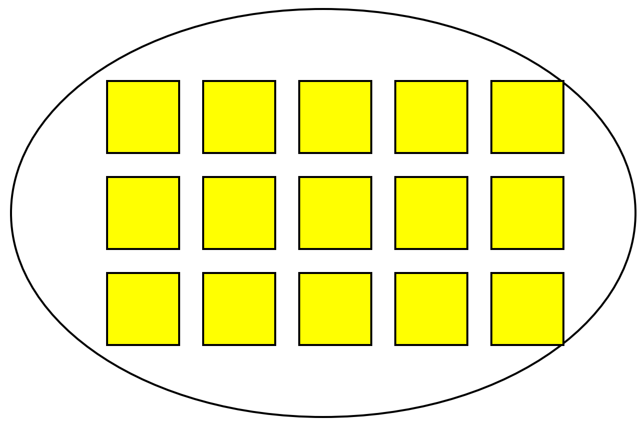
- Just like we have seen previously, we can use two different methods to multiply integers. Namely, with tiles and with number lines.

Tile Method

- Remember, a yellow tile  = +1 and a red tile  = -1.
- A zero pair is one red and one yellow tile. (+1) + (-1) = 0
- Draw a circle and think of it as a piggy bank. Start with the bank having no value. That is no tiles inside.
- The first integer in the equation tells us to either deposit (put in) or withdraw (take out). If the first integer is *positive*, put in "sets" of tiles **into** the bank. If the first tile is *negative*, take tiles "sets" **out** of the bank.
- The second integer tells us what (how many) to put in or take out.
- Let's get some examples to get our noodles in shape ☺

Examples:

1. (+3) x (+5) =



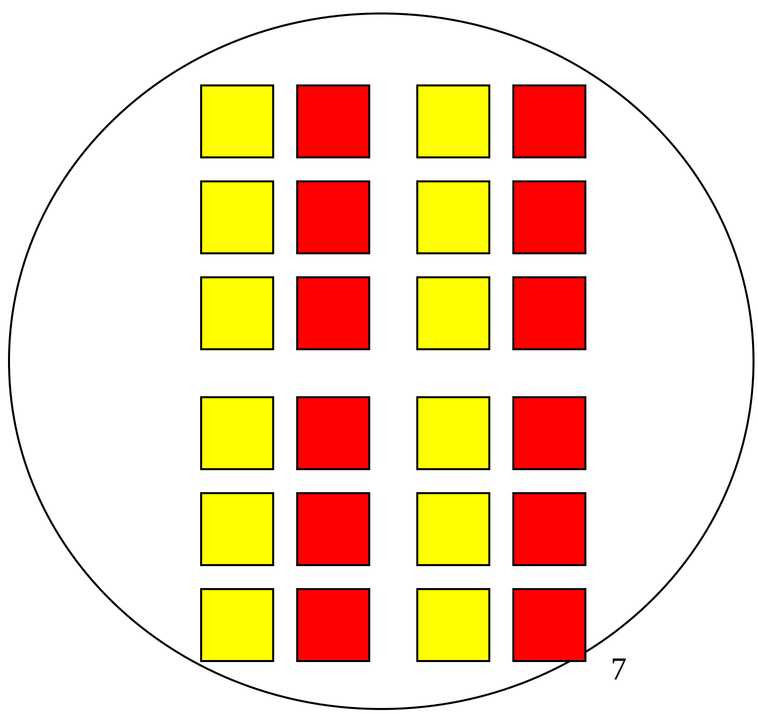
+3 is a positive integer. Thus, we will be ***putting in*** 3 "sets"…the 2nd integer tells us how many.

+5 is modeled with 5 yellow tiles.

Therefore, we will put in 3 sets of +5 tiles.

So, what do we have?

2. (-4) x (+3) =



-4 means ***take out*** 4 sets…

+3 is modeled with 3 yellow tiles.

Therefore, we will take out 4 sets of 3 yellow tiles.

Problem, nothing in the circle. What do we do?

Add zero pairs!!! Need to keep adding Zero pairs until there are enough.

Remove four sets of +3…what's left?

Try these ones…(show the tiles ☺)

(+6) x (-3) =

(-6) x (+2) =

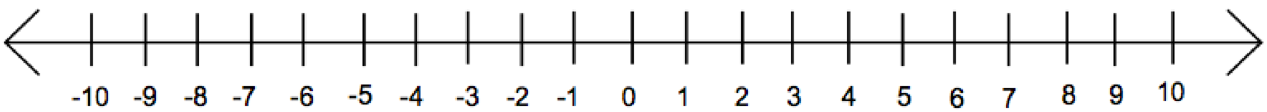
(-5) x (-4) =

Number Line Method

- Going to 'walk the number line' to multiply integers. The first integer in the equation tells you '*how many steps*'. If this integer is negative, face the negative (left side) of the number line. If this first integer is positive, face the positive (right side) of the number line.
- The second integer tells you the step size (how big one step is). If the integer is positive, walk forward, if the integer is negative, walk backwards…carefully!!!!

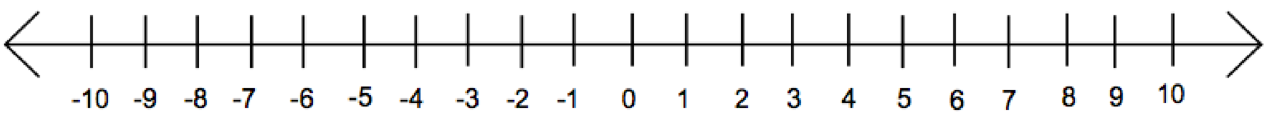
Examples:

1. (-3) x (+2) =



-3: "-ve" means face the negative end of the number line (left side). The 3 means taking 3 steps. +2: "+ve" means walk forward with a step size of 2. So, where do you end up?

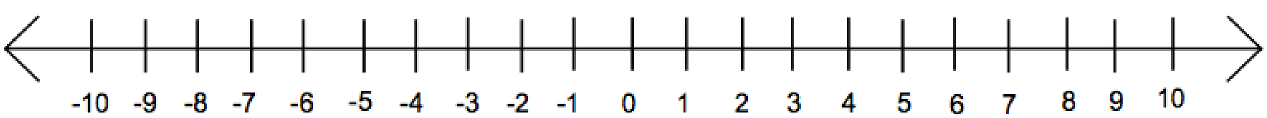
2. (-2) x (-4) =



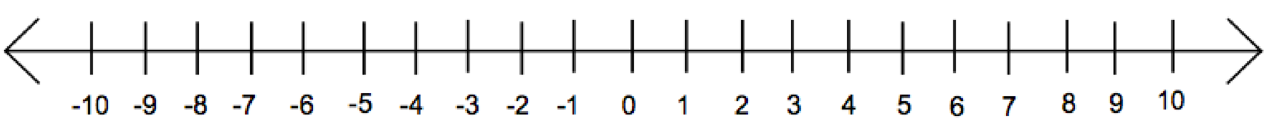
This is tricky licky!!! -2: -ve means face left, 2 means 2 steps. -4: -ve means walk backwards, 4 means a step size of 4 ☺. Again, where do you end up?

Try these:

(+3) x (+3) =



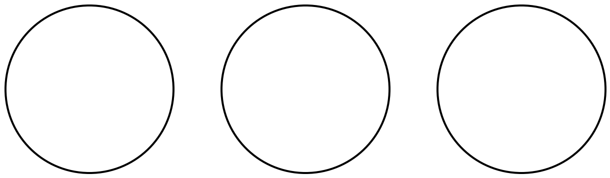
(-4) x (-1) =



Do ***multiply integers worksheet***

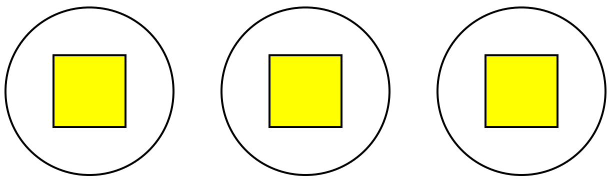
Rules for Multiplying Integers

- What happens when you multiply by zero? Look at 3 x 0, what does it mean? How about 0 x 3? (Zero property)

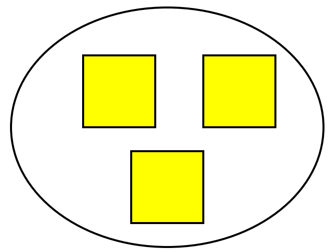


3 groups of 0

- What about multiply by 1 (Multiplicative Identity). What does (+3) x (+1) mean? Or (+1) x (+3)?



3 groups of 1



1 group of 3

Since multiplying by 1 does not change the identity of a number, we call 1 the *multiplicative identity*.

- Distributive property = very important!!

$$\begin{aligned} 3 \times (4 + 3) &= 3 \times 4 + 3 \times 3 \\ &= 12 + 9 \\ &= 21 \end{aligned}$$

How about these one?

$$\begin{aligned} (-5) \times [(+2) + (-6)] &= (-5) \times (+2) + (-5) \times (-6) \\ &= (-10) + (+30) \\ &= +20 \end{aligned}$$

$(+4) \times [(-10) + (+7)] =$

$(-10) \times [(--2) + (-5)] =$

- Look back over the notes and worksheets regarding multiplying integers. What do you notice? Any tricks?

- How do we deal with multiplying to large numbers? Such as (+20) x (-48) Any ideas?

- Oh, forgot to mention. ***Product*** means the result when two or more numbers are multiplied. For example, find the product of (-3) x (-9)

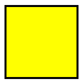
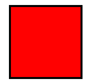
Dividing Integers (Section 2.3)

- Division can be thought of as the opposite of multiplication.

  (+15) ÷ (+3) = ? Could be changed into ? x (+3) = (+15)

- ***Quotient*** is the number that results from the division of one number by another.
- As before, we will be looking at using tiles and also a number line to divide integers.

Tile method

- Going to use the 'bank' idea, just like multiplying integers. Again, a yellow tile  = +1 and a red tile  = -1
- The first integer shown is like the product (if we switch it around to multiplication). Look at the example above. (+15) is the product of two integers.
- The second integer asks how many groups we will need to get the first integer. Alright…this is a clear as mud. Let's look at an example.

Examples:

1. Divide (-16) ÷ (+4)

   Read the expression like this…*how many groups of +4 have to be added / taken away to make (-16)*?

   Start with an empty 'bank' (zero value).

   To get (-16), we would need to have ***16 red tiles*** in the circle.

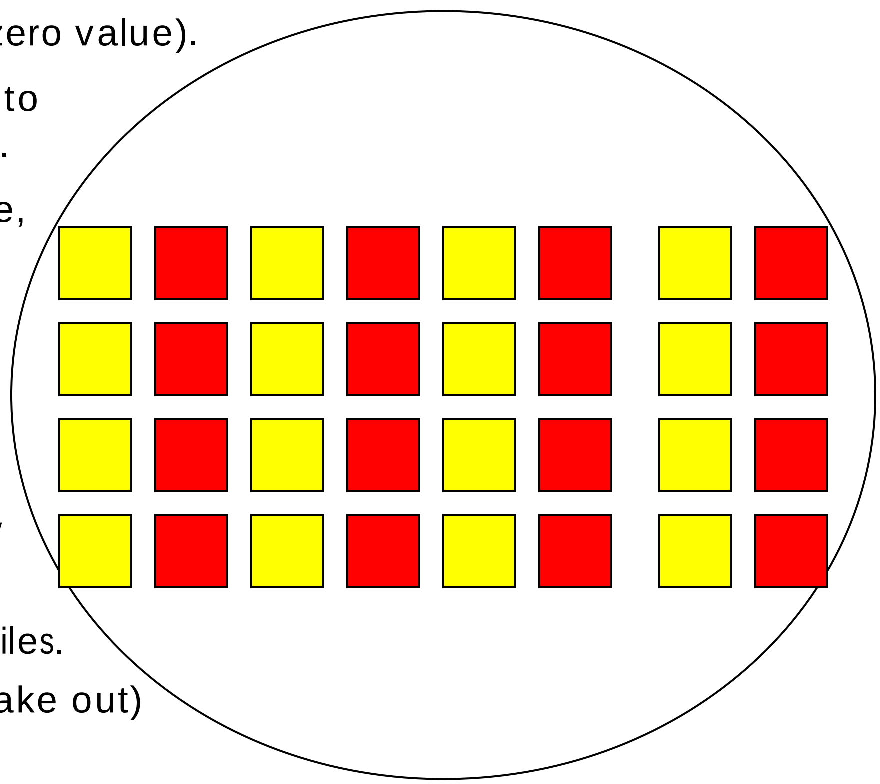
   To get 16 red tiles in the circle, add 16 zero pairs.

   In order to have 16 red tiles In the circle, what has to happen?

   (+4) is modeled with 4 yellow tiles.

   So, take out 4 sets of yellow tiles.

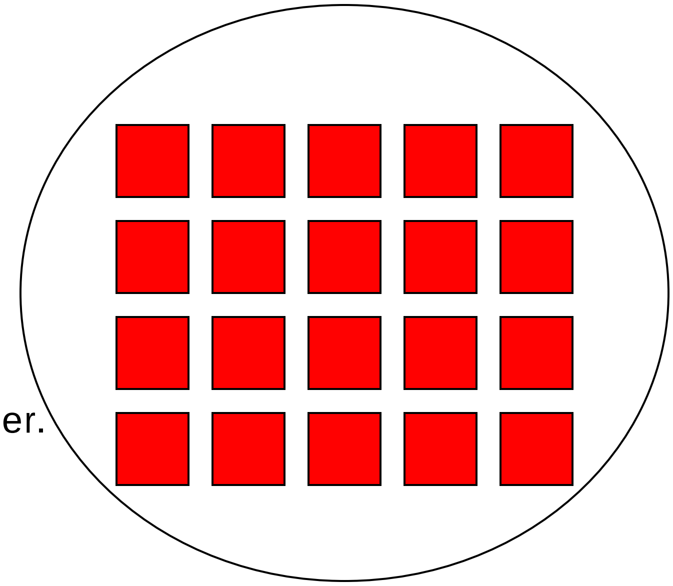
   4 sets removed. Removed (take out) means 'negative'
   So (-16) ÷ (+4) = (-4)



2. (-20) ÷ (-4) =

How many groups -4 will have to added / taken away to get -20?

(-20) means 20 red tiles. Groups of (-4), means groups of 4 red tiles.



Because we are dealing with red tiles (negative numbers) only, we can simply add groups of 4 red tiles, till we have 20 red tiles in the circle.

How many groups you add is the answer.

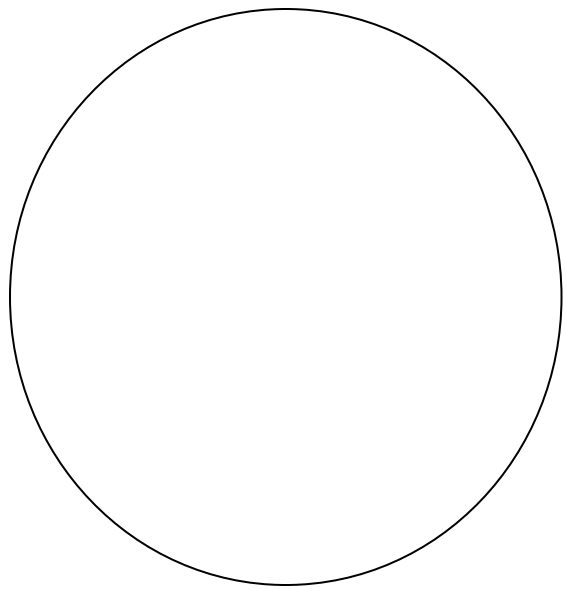
Adding into the bank means a positive answer.

We added 5 groups of -4 tiles. Therefore,

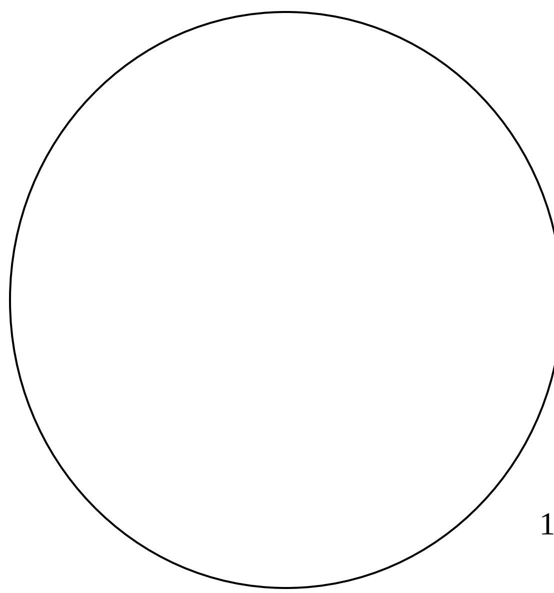
(-20) ÷ (-4) = ***+5*** (positive because we added the tile)

Try these ones...

(+18) ÷ (-2) =



(+ 24) ÷ (+6) =



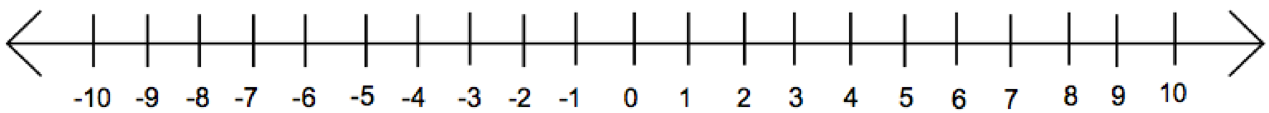
Number line Method

- Going to be ‘walking’ the number line to divide integers. In dividing, the direction we are facing determines the sign of the integer.
- Start at zero. The first integer tells you where you want to end up. The second integer tells you how big your step size is. If the step size (2nd integer) is positive, walk forward. If the step size (2nd integer) is negative walk backwards.
- Let’s look at some examples…

Examples:

1. Divide (+10) ÷ (-2) using the number line.

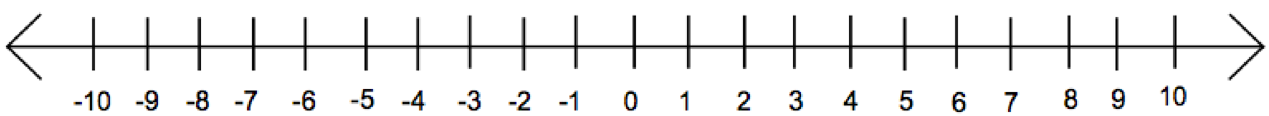
   (+10) is where we want to end up on the number line. -2 : 2 means the step size, -ve means walk backwards.



   Start at zero. To end up at +10, we would need to take 5 steps backwards. Because we are facing the negative end of the number line, the answer is negative. (+10) ÷ (-2) = (-5)

2. Divide (-8) ÷ (+2) using a number line.

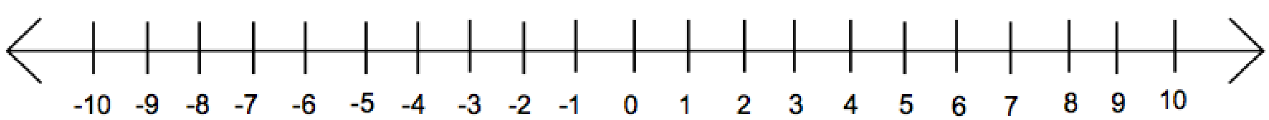
   (-8) is where we want to end up. +2 : 2 indicates the step size, +ve means walk forward.



   Start at zero. Walk forward…step size is 2…how many steps till (-8)? Which way are you facing? (-8) ÷ (+2) = _______

Try this one…

Divide (-9) ÷ (-3) using a number line



Rules for Dividing Integers

- Again, go back over the above examples…notice anything cool and awesome?

- Dividing is the ***inverse*** (opposite) of multiplying.

$(+12) \div (-3) = ?$ Could be changed into $? \times (-3) = (+12)$

dividend divisor quotient

- A division equation can be written with a division sign or as a fraction

$$(+12) \div (-3) \quad \text{or} \quad \frac{+12}{-3}$$

- When the expression is written as a fraction, we *DO NOT* have to use brackets. The fraction bar (line) separates the integers.

Order of Operations with Integers (Section 2.5)

- Look at the following expression, what's the answer?

$$12 + 9 \div 3 - 3 \times 4$$

- To ensure everyone gets the same answer, we need rules to know which operations to do first. These 'rules' are called the ***order of operations***.

- Use the expression "BEDMAS" to remember the order.

B = **B**rackets
E = **E**xponents
D = **D**ivision
M = **M**ultiplication
A = **A**ddition
S = **S**ubtraction

In this section, we will not be concerned with exponents. Thus, you will do brackets first, 'skip' exponents, and go on to divide / Multiple and then Add / Subtract.

- ***Important*** Just because divide appears first, does not mean it has priority over multiplication. Same with Addition over subtraction.
- Think of division and multiplication as being ***equal***. Do them in order from ***left to right***.
- Same with addition and subtraction. Again, they are ***equal***. Do them in order from ***left to right***.

- Because brackets ( ) are use around integers to contain the signs, square brackets [ ] are used to group terms.

- Recall that a fraction bar indicates division. Also, make sure you do the operation in the numerator (above the bar) and denominator (below the bar) before you divide. The fraction bar acts like a set of square brackets.

Examples:

1. $[(-5) + (-1)] \div [(-9) + (+6)]$

2. $\dfrac{(-8) + (+6) \times (+3)}{(-15) \div (-3)}$

3. $\dfrac{(+27) \div (-3) + (+29)}{(+5) \times (+3) + (-10)}$